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A mathematical analysis of a membrane bioreactor containing a sludge disintegration system

Mark Nelson
University of Wollongong, mnelson@uow.edu.au

Thomas Yue
University of Wollongong, tcly313@uow.edu.au

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Abstract

The activated sludge process is widely used to treat domestic and industrial wastewater. A significant drawback of this process is the production of "sludge", the disposal of which can comprise a significant proportion of the total operating costs of a wastewater treatment plant. We analyze the steady-state operation of a membrane bioreactor system (MBR) incorporating a sludge disintegration unit (SDU) to reduce sludge production. We provide a qualitative understanding of the model by finding analytically the steady-state solutions of the model and determining its stability as a function of the residence time. In practice a target value of the mixed liquor suspended solids (MLSS) content within the membrane reactor is specified. Applying the mathematical technique of singularity theory we show that if the sludge disintegration factor is sufficiently high then the MLSS content is guaranteed to be below the target value. This model prediction, of key interest from a practical perspective, was not identified in the original investigation of this model, which relied upon numerical integration of the governing equations.

Keywords

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A mathematical analysis of a membrane bioreactor containing a sludge disintegration system

M.I. Nelson* & Thomas C.L. Yue
School of Mathematics and Applied Statistics, University of Wollongong,
Wollongong, NSW 2522 Australia

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Abstract

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We analyze the steady-state operation of a membrane bioreactor system (MBR) incorporating a sludge disintegration unit (SDU) to reduce sludge production. We provide a qualitative understanding of the model by finding analytically the steady-state solutions of the model and determining their stability as a function of the residence time.

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*Corresponding author. School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, NSW 2522 Australia. Telephone: +61-2-4221-440. Fax: +61-2-4221-4845. Email: nelsonm@member.ams.org.

1 Introduction

The most widely used biological wastewater treatment method for both domestic and industrial wastewaters is the activated sludge process (Wei *et al*, 2003). At its simplest this process consists of a continuous flow bioreactor through which wastewater flows. Microorganisms within the reactor grow through the consumption of pollutants (substrate) within the wastewater to produce more microorganisms and a variety of products which typically contain carbon dioxide, nitrogen, water and other species, including biological compounds, specific to the process under consideration.

Membrane bioreactors, which couple the biological treatment process of the activated sludge process with membrane technology to physically retain microorganisms inside the reactor, have been an alternative to the conventional activated sludge process since the late 1960s (Yoo *et al*, 2004b). The membrane enhanced substrate removal by increasing the concentration of microorganisms inside the bioreactor. Furthermore, ultrafiltration membranes retain solids and high-molecular weight compounds within the reactor that can not be removed completely from the effluent stream of a conventional activated sludge reactor. This further improves the quality of the water delivered by a membrane reactor over that emerging from a conventional reactor. In addition to the higher quality effluent further advantages of membrane bioreactors include their compactness, short start-up times, and lower operating and maintenance manpower requirements (Yang *et al*, section 3.3, 2006).

In view of their advantages membrane bioreactors are increasingly used as key elements of advanced wastewater processing schemes to treat domestic, industrial and specific municipal wastewater (Yang *et al*, 2006). Due to their compactness membrane bioreactors are particularly suited for the development of wastewater treatment facilities in urban areas (Wisniewski, 2007).

A significant drawback of the activated sludge process, both in a membrane bioreactor or a continuous flow bioreactor, is the production of 'sludge'. The expense for treating excess sludge can account for 50–60% of the total operating costs in a wastewater treatment plant (Canales & Poles, 1994; Egemen *et al*, 2001; Nowak, 2006). Traditional methods for disposing of excess sludge, such as incineration, the use of landfill sites, and dumping at sea, are becoming increasingly regulated in many countries due to environmental concerns about the presence of potentially toxic elements. Furthermore, a combination of the limited amount of land available for landfill, particularly in urban areas, with increasingly stringent legislation has resulted in the economic costs of using landfill sites raising sharply. It should be noted that incineration does not eliminate the need for landfill sites as a product of incineration is an ash containing high heavy materials content and general toxicity. Thus there is a pressing need, and growing interest, in methods that reduce the volume and mass of excess sludge produced as part of biological wastewater treatment processes.

Although membrane reactors produce less sludge than the conventional activated sludge process, sludge must still be removed to maintain appropriate levels of biomass within the reactor. Indeed the performance of the membrane may deteriorate at high values of the MLSS (mixed liquor suspended solids) (Yamamoto *et al* 1989,

Chang *et al* 2002; Le Clech *et al* 2003). The reasons for this reduction include poor oxygenation (which increases aeration costs) and extensive membrane fouling (which requires frequent membrane cleaning and replacement) (Wei *et al*, section 4.1, 2003).

Excess sludge production, in both conventional and membrane bioreactors, can be reduced by increasing sludge biodegradability by disintegrating it within the reactor. This approach works primarily by disintegrating bacterial cell walls. A variety of methods have been used in the literature to achieve this aim including: the addition of anoxic/anaerobic stages (Barker & Dold, 1996; Yoon, 2003), enzyme treatment (Barjenbruch & Kopplow, 2003), freezing and thawing (Chu *et al*, 1999), the use of an inclined-plate membrane bioreactor (Xing *et al*, 2006), ozonation (Yasui *et al*, 1996; Sakai *et al*, 1997; Egemen *et al*, 2001; Song *et al*, 2003; Wang *et al*, 2008), ozonation combined with chemical treatment (Oh *el al*, 2007), thermal treatment (Goma *et al*, 1997; Kepp *et al*, 1999), and ultrasound (Yoon *et al*, 2004b). Combinations of these treatments are also possible.

An alternative approach to decrease sludge production is to increase biodegradability through the growth of controllable predators (Wei *et al*, 2003; Zhang & Yamamoto , 2006). It is also possible to decrease sludge production by increasing the proportion of energy that is released by substrate consumption that is used for non-growth activities as opposed to growth activities (Wei *et al*, 2003).

Although many different techniques to decrease sludge production have been investigated, the most widely adopted techniques in commercial activated sludge plants are chemical treatments and ozone treatments (Oh *et al*, 2007). The model considered in this paper is appropriate to the latter method. In these methods a part of the sludge is removed from the reactor and treated with ozone in a sludge disintegrator. This ozonation stage converts the live sludge into a mixture of soluble substrate and particulates. The liquidized sludge is then returned to the bioreactor as a feed solution where the soluble substrate is biodegraded by live sludge. This technique has been shown to lead to much lower MLSS levels.

In practice a target value for the MLSS is specified. If the steady-state MLSS value is below (above) the target value then the plant is said to be operating in a state of negative (positive) excess sludge production. The transitional case when the steady-state MLSS value is equal to the target value corresponds to zero excess sludge production.

In order to investigate how the process variables associated with a sludge disintegration unit (SDU) effect the steady-state MLSS value Yoon (2003) proposed a model for a membrane bioreactor containing a sludge disintegration unit (MBR-SDU). In this model the sludge disintegration unit is not modelled per se. Instead sludge disintegration terms are added to a conventional activated sludge model. These terms assume that the disintegrator unit destroys the biochemical activity of the sludge, converting a fraction, α, directly into usable substrate and the remainder, $(1-\alpha)$, into organic particulates. These processes are defined in equations (5) & (6) respectively.

Yoon investigated the model (1)–(7) by integrating the governing equations until a steady-state solution was reached (Yoon, 2003). The contribution of our paper is to re-analyse this model by directly determining

the steady-state behaviour. In so doing we obtain an improved understanding of the relationship between operational parameters and system outputs (the effluent concentration and the MLSS) than can be obtained through repeated numerical integration of the governing equations. In particular, we identify conditions which allow the MBR-SDU to operate in a condition of zero excess sludge production.

Furthermore, our knowledge of the steady-state solutions allows us to identify an important practical prediction of the model that was not realised during the initial, numerical, investigation of this model. Namely, that a MBR-SDU can be guaranteed to operate in a state of negative excess sludge production if the sludge disintegration factor (D) is sufficiently high.

1.1 Model equations

The model proposed by Yoon (2003) contains two components. The first component is a membrane bioreactor model (MBR) which consists of equations for the concentration of microorganisms, substrate and particulates within a well-stirred, well aerated, membrane bioreactor. The membrane bioreactor model is given by equations (1)–(3) The second component is a model for the sludge disintegrator unit. This is given by equations (5) & (6). Features of equations (1)–(6) are described below. Figure 1 provides a schematic of the reactor configuration, including important process variables.

In equation (1) it is assumed that the Monod parameters (μ_m and K_S) are the same for all soluble organic materials whether they originated from the influent or the disintegrated sludge. This is reasonable as biomass growth that occurs on the original organic substrate can not be distinguished from biomass growth that occurs on the disintegrated sludge (Wei *et al*, section 2, 2003). It is also assumed that organic particulates are hydrolyzed to generate soluble substrate.

The equation for the concentration of the microorganisms, equation (2), includes a term $-k_d X$ which represents a combination of first-order processes. These include cell death, cell lysis, endogenous respiration, and predation (Pavlostathis & Giraldo-Gomez, 1991). In equations (2) & (3) the term DF represents the flow of material from the reactor to the substrate disintegration unit.

Equations (5) & (6) represent the concentration of substrate and particulates flowing out of the disintegration unit and into the membrane bioreactor. It is assumed that: soluble substrate is not affected by the disintegrator; that the solubilization efficiency (α) and conversion efficiency (β) are independent of other process parameters; and, that the disintegrator unit is 100% efficient — all microorganisms entering the SDU are converted either into particles or substrate. In equation (5) the term $[\alpha\beta\,(X+P)]$ represents the disintegrated sludge produced by the disintegration unit. The efficiency of the the sludge disintegration process is represented by the solubilization ratio α.

The model equations (1)–(7) were introduced by Yoon (2003). Yoon's model was modified by Wang *et al* (2008) to analyze experimental data obtained in a bench-scale study of a membrane bioreactor coupled with an ozonation unit. In (Wang *et al*, 2008) the growth of biomass upon the substrate was not modelled by the Monod

expression, instead the amount of biomass produced through consumption of the substrate was estimated based upon the difference between the substrate concentration in the influent and effluent streams. The values of operational parameters to achieve zero excess sludge production predicted by the model were found to match well those found from the experimental study. The work reported by Wang *et al* (2008) therefore acts as experimental justification of the model proposed by Yoon (2003).

Differential equations could be written down for the concentrations of substrate, biomass and particulates within the sludge disintegration unit. In such a model the disintegration of biomass and particulates would be represented by first-order processes. Yoon's model arises in the limit when the corresponding rate constants approach infinity. In effect Yoon's model assumes that the time-scale associated with the break-up of biomass and particulates is significantly smaller than that associated with the biochemical processes; i.e. the growth of the biomass upon the substrate, 'natural' decay of the biomass and hydrolysis of the particulates. This approximation was shown to hold in the experimental work reported by Wang *et al* (2008).

1.2 Materials and Methods

Yoon's model (Yoon 2003) consists of a conventional activated sludge model for a membrane bioreactor with the addition of sludge disintegration terms. The values for the kinetic and stoichiometric parameters, provided in appendix A, were taken from a variety of literature sources, the appropriate references are provided by Yoon (2003, table 1). Yoon also provided values of operational parameters (Yoon 2003, table 2). Of these, the values for the flow rate ($F = 2\,\mathrm{L\,day^{-1}}$), the influent concentration ($S_0 = 4000\,\mathrm{mg\,L^{-1}}$) and the volume of the reactor ($V = 8\,\mathrm{L}$) came from an experimental study (Yoon *el al*, 2004b). The sludge disintegration method used in this study was ultrasound.

Wang's experimental study (Wang *et al*, 2008) confirmed Yoon's model. In this study the sludge disintegration method used was ozonation. The flow rate and the reactor volume were $F = 300\,\mathrm{L\,day^{-1}}$ and $V = 100\,\mathrm{L}$ respectively. The dissolved oxygen concentration was in the range $0.5 - 1.0\,\mathrm{g\,L^{-1}}$. The COD of the influent was $S_0 = 190 - 250\,\mathrm{mg\,L^{-1}}$.

2 Equations

2.1 The dimensional model

The model equations are

Concentration of substrate

$$V\frac{\mathrm{d}S}{\mathrm{d}t} = F\left(S_0 - S\right) + DF\left(S_r - S\right) + \beta k_h V P - \frac{\mu\left(S\right)}{Y} V X. \tag{1}$$

Concentration of biomass

$$V\frac{\mathrm{d}X}{\mathrm{d}t} = -DFX + VX\mu\left(S\right) - Vk_dX. \tag{2}$$

Concentration of particulates

$$V\frac{\mathrm{d}P}{\mathrm{d}t} = DF\left(P_r - P\right) - k_hVP. \tag{3}$$

Specific growth rate

$$\mu\left(S\right) = \frac{\mu_m S}{K_S + S}. \tag{4}$$

Concentration of recycled substrate

$$S_r = S + \alpha\beta\left(X + P\right). \tag{5}$$

Concentration of particulates (damaged biomass)

$$P_r = \left(1 - \alpha\right)\left(X + P\right). \tag{6}$$

Residence time

$$\tau = \frac{V}{F}. \tag{7}$$

The definition of the parameters in the model, and their typical values, are provided in appendix A.

For a specific wastewater, a given biological community, and a particular set of environmental conditions the parameters K_s, Y, k_d, k_h, β, and μ_m are fixed. The parameter α depends upon the performance of the disintegration process. The parameters that can be readily varied are D, S_0, and τ. The main experimental control parameter is the residence time, given in equation (7).

Consider an experiment carried out in a batch reactor in which one unit of substrate is consumed to produce Y units of biomass. The biomass is placed into a sludge disintegration unit which produces $\alpha\beta Y$ units of substrate and $\left(1 - \alpha\right)Y$ units of particulate matter. When fully hydrolyzed the particulate matter gives $\left(1 - \alpha\right)\beta Y$ units of substrate. Thus the amount of substrate produced from the consumption of one unit of substrate is βY units of substrate $[\alpha\beta Y + \left(1 - \alpha\right)\beta Y = \beta Y]$. Conservation of mass therefore imposes the restriction that $\beta Y \leq 1$. A similar observation was made by Wang *et al* (2008).

As noted in the introduction a quantity of particular interest in the study of this system is the steady-state value for the mixed liquor suspended solids (MLSS). This is equivalent to the concentration of sludge within the reactor and is defined by

$$\mathrm{MLSS} = X + P.$$

Henceforth whenever we discuss MLSS it is implied that this is measured under steady-state conditions.

2.2 The dimensionless model

By introducing dimensionless variables for the substrate concentration $[S^* = S/K_s]$, the cell mass concentration $[X^* = X/(YK_s)]$, the particulate concentration $[P^* = P/(YK_s)]$, and time $[t^* = \mu_m t]$ the dimensional model, equations (1)–(3), can be written in the dimensionless form

$$\frac{\mathrm{d}S^*}{\mathrm{d}t^*} = \frac{1}{\tau^*}(S_0^* - S^*) + \frac{\alpha\beta^* D}{\tau^*}(X^* + P^*) + \beta^* k_h^* P^* - \frac{S^*}{1+S^*}X^*, \tag{8}$$

$$\frac{\mathrm{d}X^*}{\mathrm{d}t^*} = \left[-\frac{D}{\tau^*} + \frac{S^*}{1+S^*} - k_d^*\right]X^*, \tag{9}$$

$$\frac{\mathrm{d}P^*}{\mathrm{d}t^*} = \frac{D}{\tau^*}[(1-\alpha)X^* - \alpha P^*] - k_h^* P^*, \tag{10}$$

where the parameter groups are: the dimensionless substrate concentration in the feed $[S_0^* = S_0/K_s]$, the dimensionless sludge decay coefficient $[k_d^* = k_d/\mu_m]$, the dimensionless hydrolysis coefficient $[k_h^* = k_h/\mu_m]$, the dimensionless particulate to substrate conversion factor $[\beta^* = Y\beta]$ and the dimensionless residence time $[\tau^* = V\mu_m/F]$. All parameters in the model are strictly non-negative. Furthermore, we assume that $S_0^* > 0$ and $k_d^* > 0$. The restriction $\beta Y \leq 1$ becomes $\beta^* < 1$. There is a one-to-one relationship between our dimensionless variables and their dimensional counterparts. Hence we often write, for example, 'sludge decay coefficient' rather than 'dimensionless sludge decay coefficient'.

Typical values of the dimensionless parameters, used for calculations, are stated in appendix A.

The scaled value of the mixed liquor suspended solids (MLSS^*) is given by

$$\mathrm{MLSS}^* = X^* + P^*.$$

3 Results

In section 3.1 the steady-state solution branches are given and the condition for the no-washout solution branch to be physically meaningful is identified. The steady-state value for MLSS along the washout branch is given. In section 3.2 the stability of the steady-state solutions is determined. In section 3.3 asymptotic solutions for large residence times are stated.

3.1 Steady-state solution branches

The steady-state solutions are given by

Washout branch

$$(S^*, X^*, P^*) = (S_0^*, 0, 0)\,, \tag{11}$$

No-washout branch

$$(S^*, X^*, P^*) = \left(\hat{S}^*, \frac{S_0^* - \hat{S}^*}{(1-\beta^*)\,D + k_d^*\tau^*}, \frac{(1-\alpha)\,D}{\alpha D + k_h^*\tau^*} \cdot X^*\right), \tag{12}$$

$$\hat{S}^* = \frac{D + k_d^*\tau^*}{(1-k_d^*)\,\tau^* - D}. \tag{13}$$

The value for the mixed liquor suspended solids simplifies to

$$\mathrm{MLSS}^* = \left[\frac{D + k_h^*\tau^*}{\alpha D + k_h^*\tau^*}\right] X^*. \tag{14}$$

The no-washout branch is only physically meaningful when the substrate, cell-mass, and particulate concentrations are positive ($S^* > 0, X^* > 0, P^* > 0$). This happens when ($0 \le \beta^* < 1$)

$$\begin{aligned} \tau^* > \tau_{\mathrm{tr}}^* &= \frac{(1+S_0^*)\,D}{S_0^* - (1+S_0^*)\,k_d^*}, \\ 0 < k_d^* &< \frac{S_0^*}{1+S_0^*}. \end{aligned} \tag{15}$$

3.2 Stability of the steady-state solutions

The Jacobian matrix is given by

$$J\,(S^*, X^*, P^*) = \begin{pmatrix} -\frac{1}{\tau^*} - \frac{X^*}{\left(1+S^{*2}\right)} & \frac{\alpha\beta^* D}{\tau^*} - \frac{S^*}{1+S^*} & \frac{\alpha\beta^* D}{\tau^*} + \beta^* k_h^* \\ \frac{X^*}{(1+S^*)^2} & \frac{-D}{\tau^*} + \frac{S^*}{1+S^*} - k_d^* & 0 \\ 0 & \frac{(1-\alpha)D}{\tau^*} & -\frac{\alpha}{\tau^*} - k_h^* \end{pmatrix}.$$

The Jacobian matrix evaluated at the washout steady-state solution is given by

$$J\,(S^*, X^*, P^*) = \begin{pmatrix} -\frac{1}{\tau^*} & \frac{\alpha\beta^* D}{\tau^*} - \frac{S_0^*}{1+S_0^*} & \frac{\alpha\beta^* D}{\tau^*} + \beta^* k_h^* \\ 0 & \frac{-D}{\tau^*} + \frac{S_0^*}{1+S_0^*} - k_d^* & 0 \\ 0 & \frac{(1-\alpha)D}{\tau^*} & -\frac{\alpha D}{\tau^*} - k_h^* \end{pmatrix}.$$

The eigenvalues of this matrix are

$$\begin{aligned} \lambda_1 &= -\frac{1}{\tau^*} < 0, \\ \lambda_2 &= -\frac{\alpha D}{\tau^*} - k_h^* < 0, \\ \lambda_3 &= \frac{-D}{\tau^*} + \frac{S_0^*}{1+S_0^*} - k_d^*. \end{aligned}$$

It follows that the washout branch is always stable if

$$k_d^* \ge \frac{S_0^*}{1+S_0^*}.$$

If $k_d^* < \frac{S_0^*}{1+S_0^*}$ then the washout steady-state is stable provided

$$\tau^* < \tau_{\rm tr}^* = \frac{(1+S_0^*)\,D}{S_0^* - (1+S_0^*)\,k_d^*}.$$

We have shown that the washout solution is stable when $\tau^* < \tau_{\rm tr}^*$. For such values, the no-washout solution is not physically meaningful – see (15). It is to be expected that when the washout solution is physically meaningful that it is stable. We show that this is true in appendix B.

A transcritical bifurcation occurs, as the residence time is varied, when

$$\tau^* = \tau_{\rm tr}^* = \frac{(1+S_0^*)\,D}{(1-k_d^*)\,S_0^* - k_d^*}. \tag{16}$$

At this value of the residence time the no-washout solution branch and the washout solution branch intersect at the point

$$(S^*, X^*, P^*, \tau^*) = (1, 0, 0, \tau_{\rm tr}^*)\,.$$

The value of the residence time represents the maximum residence time at which the treatment process fails. At lower residence times microorganisms are removed from the system at a rate greater than their maximum growth rate, resulting in process failure. At residence times lower (higher) than the transcritical value the washout (no-washout) solution is the only stable solution. Equation (16) shows that the washout condition only depends upon the operation of the sludge disintegration system through the value of the sludge disintegration factor (D).

3.3 Large residence time approximations

At large residence times we have the approximations ($D \neq 0$)

$$S^* \approx \frac{k_d^*}{1-k_d^*} + \frac{D}{(1-k_d^*)^2}\cdot\frac{1}{\tau^*} + O\left(\frac{1}{\tau^{*^2}}\right), \tag{17}$$

$$X^* \approx \frac{(1-k_d^*)\,S_0^* - k_d^*}{(1-k_d^*)\,k_d^*}\cdot\frac{1}{\tau^*} + O\left(\frac{1}{\tau^{*^2}}\right), \tag{18}$$

$$P^* \approx \frac{(1-\alpha)\,D}{k_h^* k_d^*}\left(S_0^* - \frac{k_d^*}{1-k_d^*}\right)\cdot\frac{1}{\tau^{*^2}} + O\left(\frac{1}{\tau^{*^3}}\right), \tag{19}$$

$$\mathrm{MLSS}^* \approx \frac{(1-k_d^*)\,S_0^* - k_d^*}{(1-k_d^*)\,k_d^*}\cdot\frac{1}{\tau^*} + O\left(\frac{1}{\tau^{*^2}}\right). \tag{20}$$

At large residence times the effluent and microorganism concentrations are, to leading order, independent of the operation of the sludge disintegration unit. At sufficiently high residence times the use of a disintegration unit increases the effluent concentration slightly. Note that the concentration of particulates decreases much faster at high residence times than the concentrations of the substrate and microorganism. In these formulae the case $k_d^* = 1$ is not allowed, as when $k_d^* = 1$ the no-washout branch is not physically meaningful; see section 3.1.

Equation (17) shows that at high residence times there is a non-zero limiting concentration, $S^* = \frac{k_d^*}{1-k_d^*}$, below which the effluent concentration in a membrane bioreactor with a sludge disintegration unit can not be reduced. The limiting value is independent of the operation of the sludge disintegration unit. This is a consequence of the assumption that the specific growth rate is independent of the biomass concentration.

4 Discussion

In this section we discuss the implications of the results obtained in section 3.

In section 4.1 we consider the steady-state values for the concentration of the effluent, biomass, particulates and the mixed liquor suspended solids content. We demonstrate how knowledge of the analytical expressions for the steady-state solutions allows a greater insight into the relationship between the process parameters and the system outputs than is obtained by numerical integration of the governing differential equations.

In section 4.2 we investigate how the values of the residence time corresponding to zero excess sludge production depend upon the sludge disintegration factor. We show that there is a critical value of the sludge disintegration factor above which the system is guaranteed to operate in a state of negative sludge production, i.e. the operating value of the MLSS is always below the target value. Using the known expressions for the steady-state values it is straightforward to determine this critical value.

In section 4.3 we evaluate two ways of defining the specific utilization of the reactor. This section again shows the gains to be made by using the analytical expressions for the steady-state solutions rather than relaying upon numerical integration of the governing equations.

4.1 Steady-state values

The steady-state equations (12) & (13) show that the substrate concentration is independent of the solubilization efficiency (α), the conversion efficiency from non-biodegradable particulates to biodegradable substrate (β^*), and the hydrolysis rate (k_h^*). It is an increasing function of the sludge disintegration factor D and a decreasing function of the residence time (τ^*), i.e. the lowest effluent concentration is obtained at an infinite residence time. This dependence of the effluent concentration upon the sludge disintegration factor reflects the fact that the sludge disintegration unit increases the food supply whilst decreasing the amount of live biomass. The dependence of the effluent concentration upon the sludge disintegration factor was obtained previously by integrating the governing equations for two values of D ($D = 0$ and $D = 0.5$) (Yoon, 2003).

In (Yoon, 2003) it was noted that the value of the effluent concentration was "susceptible" to changes in the kinetic parameters for biodegradability. This result was obtained by integrating the governing equations for three pairs of parameter values: ($\mu_m = 1.0\,\mathrm{day}^{-1}$, $K_s = 200\,\mathrm{mg\ COD\,L}^{-1}$), ($\mu_m = 3.0\,\mathrm{day}^{-1}$, $K_s = 100\,\mathrm{mg\ COD\,L}^{-1}$), and ($\mu_m = 6.0\,\mathrm{day}^{-1}, K_s = 20\,\mathrm{mg\ COD\,L}^{-1}$). Converting our expression for S^* into dimensionless units and using the default parameter values from appendix A we find that the steady-state effluent concentration is given by

$$S\left(\mathrm{mg\,L}^{-1}\right) = \frac{0.612K_s}{4\mu - 0.612}.$$

According to Yoon (2003) the 'reasonable range' of biodegradability is $10 \leq K_s\left(\mathrm{mg\,L}^{-1}\right) \leq 200$ and $1 \leq \mu_m\left(\mathrm{day}^{-1}\right) \leq 10$. Over this region the maximum and minimum values for the effluent concentration are given by $36.1\,\mathrm{mg\,L}^{-1}$ and $0.3\,\mathrm{mg\,L}^{-1}$. These occur for the parameter values ($\mu_m = 1.0\,\mathrm{day}^{-1}$, $K_s = 200\,\mathrm{mg\ COD\,L}^{-1}$)

and ($\mu_m = 10.0\,\mathrm{day}^{-1}$, $K_s = 20\,\mathrm{mg\ COD\,L}^{-1}$) respectively. The maximum and minimum values correspond to effluent chemical oxygen demands that have been reduced to 0.9% and 0.008% of the influent chemical oxygen demand ($S_0 = 4000\,\mathrm{mg\,L}^{-1}$).

Figure 2 (a) shows the dependence of the effluent concentration upon the residence time for three values of the sludge disintegration factor. Also plotted are the corresponding asymptotic solutions given by equation (17) — these solutions are only plotted for values of the residence time greater than the washout value given by equation (16). The accuracy of the asymptotic solutions increases with decreasing values of the sludge disintegration factor. However, for all three values the asymptotic solution is close to the exact solution when the dimensionless residence time is $\tau^* = 5$. This corresponds to a residence time $\tau = 5$ (days). By way of comparison, Yoon used a residence time $\tau = 4$ (days) in his simulations.

From equations (12) & (13) we observe that the steady-state biomass concentration is independent of the solubilization efficiency (α) and the hydrolysis rate (k_h^*). This was previously noted based upon two numerical integrations of the governing equations (Yoon, 2003). Furthermore, it is a decreasing function of the sludge disintegration factor (D) and an increasing function of the conversion efficiency from non-biodegradable particulates to biodegradable substrate (β^*).

From equations (12) & (13) we see that the particulate concentration is a decreasing function of the solubilization efficiency (α) and the hydrolysis rate (k_h^*). Thus the particulate concentration is high when the solubilization efficiency is low. It is an increasing function of the conversion efficiency from non-biodegradable particulates to biodegradable substrate (β^*).

Equation (14) shows that the mixed liquor suspended solids is an increasing function of the dimensionless particulate to substrate conversion factor (β^*) and a decreasing function of both the solubilization ratio (α) and the hydrolysis rate (k_h^*). In (Yoon, 2003) these results for the dependence of MLSS upon the parameters α and k_h^* were suggested by limited numerical integration of the governing equations.

Yoon (2003) investigated how the MLSS value depends upon the biodegradability of organic materials by numerical integration of the using governing equations for the cases ($\mu_m = 6.0\,\mathrm{day}^{-1}, K_s = 20\,\mathrm{mg\ COD\,L}^{-1}$) and ($\mu_m = 1.0\,\mathrm{day}^{-1}, K_s = 200\,\mathrm{mg\ COD\,L}^{-1}$). The values for the MLSS for these two scenarios are $8382\mathrm{mg\,L}^{-1}$ and $8307\mathrm{mg\,L}^{-1}$ respectively. On the basis of these two calculations it was concluded that the steady-state MLSS values are "almost the same irrespective of the biodegradability of organic materials if the biodegradability is in a reasonable range" (Yoon 2003, page 1925).

Converting our expression for MLS* into dimensional units and using the default parameter values from appendix A we obtain

$$\mathrm{MLSS}\left(\mathrm{mg\,L}^{-1}\right) \approx 2.1\left(4000 - \frac{0.612K_s}{4\mu - 0.612}\right).$$

Over the 'reasonable range' of biodegradability stated earlier the maximum and minimum values for the MLSS are $8382\mathrm{mg\,L}^{-1}$ and $8307\mathrm{mg\,L}^{-1}$. These are given by the parameter values ($\mu_m = 10.0\,\mathrm{day}^{-1}, K_s = 20\,\mathrm{mg\ COD\,L}^{-1}$) and ($\mu_m = 1.0\,\mathrm{day}^{-1}, K_s = 200\,\mathrm{mg\ COD\,L}^{-1}$) respectively. Thus we have rigorously estab-

lished that, over the specified parameter range, the biodegradability kinetic parameters are not a critical factor in the operation of the MBR-SDU system.

Figure 2 (b) shows the dependence of the scaled MLSS upon the residence time for three values of the sludge disintegration factor. Also plotted is the asymptotic solution, which is not as accurate as that for the effluent concentration, which to order $(1/\tau^*)$ is given by equation (18). Note that there is only one asymptotic solution in this figure because to leading order equation (20) is independent of the sludge disintegration factor. The MLSS of the exact steady-state solutions is seen to be a decreasing function of the sludge disintegration factor (D).

For values of the residence time below, or equal to, the transcritical bifurcation ($\tau^* \leq \tau^*_{\rm cr}$) the MLSS is equal to zero. For values of the residence time higher than the critical value ($\tau^* > \tau^*_{\rm cr}$) the MLSS content increases to a maximum value ($\mathrm{MLSS}_{\rm max}$) before decreasing asymptotically towards zero in the limit of infinite residence time. For sufficiently high values of the residence time equation (20) shows that the asymptotic value of MLSS is independent of the operation of the sludge disintegration unit. Thus even in the absence of a SDU, negative excess sludge production can be achieved by operating the MBR-SDU at sufficiently high residence time. Thus, as shown in figure 2 (b), the main purpose of the sludge disintegration unit is to achieve this performance at lower residence times.

We finish this section by noting that analytic determination of the steady-state solutions allows a much thorough analysis of the reactor performance upon the process parameters than can be determined from limited numerical integration.

4.2 Zero excess sludge production

Under normal operating conditions a target value for MLSS is specified ($\mathrm{MLSS}_{\rm target}$). If the MLSS value is higher than the target value, then the membrane bioreactor is producing excess sludge. If the MLSS value is equal to the target value the membrane bioreactor is producing zero excess sludge. If the MLSS value is lower than the target value, the membrane bioreactor is producing 'negative' excess sludge.

From the shape of the curves exhibited in figure 2 (b) we see that there are two generic possibilities. (This assumes that the sludge disintegration factor is greater than zero). If the target value is higher than the maximum value ($\mathrm{MLSS}_{\rm target} > \mathrm{MLSS}_{\rm max}$) then the membrane bioreactor always operates in a state of negative excess sludge production. If the target value is lower than the maximum value ($\mathrm{MLSS}_{\rm target} < \mathrm{MLSS}_{\rm max}$) then there are two values of the residence time, τ^*_1 and τ^*_2 ($\tau^*_2 > \tau^*_1$), such that when either $\tau = \tau^*_1$ or when $\tau = \tau^*_2$ the membrane bioreactor operates in a state of zero excess sludge production. Note that the effluent concentration is lower at the higher of these two values ($S^*(\tau^*_2) > S^*(\tau^*_1)$). If either $0 \leq \tau^* < \tau^*_1$ or $\tau^*_2 < \tau^*$ then the membrane bioreactor operates in a state of negative excess sludge production. Note that the effluent concentration is higher when the residence time is in the first range. Finally, if $\tau^*_1 < \tau^* < \tau^*_2$ then the membrane bioreactor operates in a state of positive excess sludge production. Thus, given a target value, we would choose $\tau^* \geq \tau^*_2$.

In (Yoon, 2003) the target value was taken to be $12,000\text{mg}\,\text{L}^{-1}$. This translates to a dimensionless value $\text{MLSS}^*_{\text{target}} = 240$. Figure 3 shows how the values of the residence time required for zero excess sludge production (τ_1^* and τ_2^*) depend upon the value of the sludge disintegration rate (D). As noted above, it is the upper line that is of interest as it corresponds to a lower effluent concentration. For a specified sludge disintegration factor this figure provides the residence time that is required to ensure zero excess sludge production. From figure 3 observe that there is a critical value of the sludge disintegration rate, $D_{\text{crit}} \approx 0.81$, where $\tau_a^* = \tau_2^* \approx 1.07$. For values of the sludge disintegration rate that are higher than the critical value the MB-SDU is guaranteed to be in a state of negative excess sludge production.

Note that there is an inherent conflict between a desire to minimise the effluent concentration, which requires $D = 0$, and a desire to ensure operation in a state of zero excess sludge, which requires $D > D_{\text{cr}}$ if this is to be true for any residence time. We first consider a scenario in which the sludge disintegration unit is turned off ($D = 0$). If the membrane bioreactor operates at a typical residence time $\tau^* = 4$ then, using the default parameter values, we find that whilst the effluent concentration is 99.3% clean, $S^* = 0.0288$, there is positive excess sludge production, $\text{MLSS}^* = 357$.

Turning the sludge disintegration unit on and setting $D = 1 > D_{\text{cr}}$, we find that the effluent concentration increases by an order of magnitude to $S^* = 0.395$. The decrease in the effluent quality is not so dramatic, it is 99.0% clean. However, the system is now operating in a state of negative excess sludge production with $\text{MLSS}^* = 122$. In practice, the 'correct' choice for the sludge disintegration factor will be determined by a variety of factors including whether there is also a target value for the effluent concentration.

The zero excess sludge production line in figure 3 is found by solving the equation

$$\mathcal{G}\left(\tau^*, D\right) = \text{MLSS}^* - \text{MLSS}^*_{\text{target}} = 0.$$

If we view figure 3 as showing the variation in the residence time as the sludge disintegration factor than the critical value of the sludge disintegration factor corresponds to a limit-point bifurcation. Thus, the critical value is found by solving the simultaneous equations

$$\mathcal{G}\left(\tau^*, D\right) = 0, \tag{21}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau^*}\mathcal{G}\left(\tau^*, D\right) = 0, \tag{22}$$

subject to the non-degeneracy condition

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^{*^2}}\mathcal{G}\left(\tau^*, D\right) \neq 0. \tag{23}$$

The existence of a critical value for the sludge disintegration factor, above which the MBR-SDU is guaranteed to operate in a state of negative excess sludge production, was not found in the earlier investigation (Yoon, 2003). which relied upon numerical integration of the governing equations.

The use of the exact values for the steady-state solution to formulate the problem of finding the critical value for the sludge disintegration factor as a singularity problem, through the solution of equations (21) & (22), shows the advantages of analytical methods to find and interpret the solutions over numerical integration.

4.3 Specific utilization

The results stated in this section only apply when the no-washout branch is physically meaningful. We require

$$\tau^* > \tau^*_{\rm tr} = \frac{(1+S_0^*)\,D}{(1-k_d^*)\,S_0^* - k_d^*}.$$

There are a number of definitions which are used to characterize the steady-state performance of a continuous flow bioreactor processing industrial wastewaters (Lawrence & McCarty, 1970). The dimensionless specific utilization ($\mathcal{U}$), which is also known as the process loading factor, the substrate removal velocity or the food to microorganism ratio, is the rate of substrate utilization per unit mass of microorganisms and is defined by

$$\mathcal{U} = \frac{S_0^* - S^*}{X^*} \cdot \frac{1}{\tau^*}.$$

Using equations (12) and (13) we obtain the specific utilization as

$$\mathcal{U} = k_d^* + \frac{(1-\beta^*)\,D}{\tau^*}. \tag{24}$$

It is known that high values for $\mathcal{U}$ may cause a flux decline in the MBR. We see that $\mathcal{U}$ is a decreasing function of the residence time, with limiting value k_d^*, and that it is independent of the organic loading (S_0^*).

Yoon (2003) suggested that when a sludge disintegration unit is used the dimensionless specific utilization should be determined using the formula

$$\mathcal{U}_{\rm sludge} = \frac{1}{X^*}\left[\frac{S_0^* - S^*}{\tau^*} + \frac{\alpha\beta^* R}{\tau^*}\,(X^* + P^*) + \beta^* k_h^* P^*\right]. \tag{25}$$

The reason for this is that in a MBR-SDU extra substrate is supplied to the reactor as disintegrated sludge and by hydrolysis of particulates. Yoon discovered by numerical integration that $\mathcal{U}_{\rm sludge}$ is independent of the solubilization efficiency (α) and the rate of hydrolysis (k_h^*). Using equations (12) and (13) $\mathcal{U}_{\rm sludge}$ simplifies to

$$\mathcal{U}_{\rm sludge} = k_d^* + \frac{D}{\tau^*}, \tag{26}$$

which increases linearly with the sludge disintegration factor. This was noted in (Yoon, 2003) on the basis of repeated numerical integrations of the governing equations.

5 Conclusion

In this paper we have re-investigated a model for a membrane bioreactor connected to a sludge disintegration system that was proposed by Yoon (2003) and validated by Wang *et al* (2008). Whereas Yoon (2003) integrated the governing differential equations numerically to find the steady-state solutions for a particular set of parameter values we have found explicit expressions for the steady-state solutions.

Yoon (2003) reported a limited investigation into the sensitivity of the (numerically determined) steady-state solution to changes in the values of the kinetic parameters for biodegradability. We have used analytical

techniques to determine the exact parametric sensitivity of the steady-state solutions. In particular, we have characterised precisely the sensitivity of the effluent concentration to changes in the biodegradability parameters whilst at the same time rigorously showing that the MLSS values are insensitive to these values.

Our analytical steady-state expressions are also used to evaluate how reactor performance depends upon process parameters. In so doing we have established some conjectures made in Yoon (2003). We showed that the effluent concentration is an increasing function of the sludge disintegration factor (D) and is independent of three parameters: the sludge solubilization ratio (α), the dimensionless conversion efficiency from non-biodegradable particulates to biodegradable substrate (β^*) and the hydrolysis rate (k_d^*). We showed that the MLSS concentration is an increasing function of the dimensionless conversion efficiency from non-biodegradable particulates to biodegradable substrate (β^*) and a decreasing function of both the solubilization ratio (α) and the hydrolysis rate (k_h^*).

The most important finding of our work involves the application of our steady-state analysis to determine the zero excess sludge production curve as a function of the two key experimental variables: the dimensionless residence time (τ^*) and the sludge disintegration factor (D). This curve deliminates regions in which 'excess' sludge and 'negative' excess sludge are produced. The former region is undesirable.

The construction of the zero excess sludge production curve enables the identification of a crucial theoretical prediction, hitherto unexpected. Namely, there is a critical value for the sludge disintegration factor, $D_{\rm crit}$. Above this critical value, for any value of the residence time above the washout value, the sludge produced in the reactor is below the target value.

We showed how to evaluate this critical value ($D_{\rm cr}$) by solving two simultaneous equations which are readily formulated in terms of our steady-state analyses. This important feature of the model was not identified in the original investigation. This shows the important of carrying out steady-state analysis, rather than relying solely on direct numerical integration of the governing equations.

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A Symbols used

In defining the model equations the units that the concentration of the substrate species, S, is measured in are denoted by $|S|$. Similarly, the units used to measure the concentrations of the microorganisms, X, and the particulates, P, are denoted by $|X|$.

D	Sludge disintegration factor.	(—)
	$0 \leq D$	
F	Flowrate through the bioreactor.	$\left(\mathrm{dm}^3\,\mathrm{hr}^{-1}\right)$
K_s	Monod constant.	$(\lvert S\rvert)$
P	Concentration of particulates.	$(\lvert X\rvert)$
P_r	Concentration of particulates in the returned sludge.	$(\lvert X\rvert)$
S	Substrate concentration in the bioreactor.	$\lvert S\rvert$
S_0	Substrate concentration in the feed.	$\lvert S\rvert$
S_r	Concentration of substrate in the returned sludge.	$\lvert S\rvert$
V	Volume of the bioreactor.	$\left(\mathrm{dm}^3\right)$
X	Concentration of biomass.	$(\lvert X\rvert)$
Y	The yield factor.	$\left(\lvert X\rvert \lvert S\rvert^{-1}\right)$
k_d	Sludge decay rate constant.	$\left(\mathrm{hr}^{-1}\right)$
k_h	Hydrolysis rate of the particulates.	(hr^{-1})
t	Time.	(hr)
α	Sludge solubilization efficiency.	(–)
	$0 \leq \alpha \leq 1$	
β	Conversion efficiency from non-biodegradable particulates to biodegradable substrate.	$\left(\lvert S\rvert (X)^{-1}\right)$
$\mu(S)$	Specific growth rate model.	$\left(\mathrm{hr}^{-1}\right)$
μ_m	Maximum specific growth rate.	$\left(\mathrm{hr}^{-1}\right)$
τ	Residence time.	(hr)
τ^*_{tr}	The value of the dimensionless residence time at the transcritical bifurcation. $\tau^*_{\mathrm{tr}} = \frac{(1+S_0^*)D}{S_0^* - \left(1+S_0^*\right)k_d^*}$	(—)

Yoon (2003) provided typical values of kinetic and stoichiometric parameters appropriate for the activated sludge process. These values were obtained from a variety of literature sources, the relevant sources are provided in (Yoon, 2003). These values are: $D = 0 - 1.25\,(0.5)$, $F = 2\,\mathrm{L\,day}^{-1}$, $K_s = 10 - 200\,(100)\,\mathrm{mg\,L}^{-1}$, $S_0 = 4000\,\mathrm{mg\,L}^{-1}$, $V = 8\,\mathrm{L}$, $Y = 0.5\,\mathrm{mg\ MLSS\ (mg\ COD)}^{-1}$, $k_d = 0.028\,\mathrm{day}^{-1}$, $k_h = 0.06 - 2.2\,(0.3)\,\mathrm{day}^{-1}$. $\alpha = 0.1 - 0.7\,(0.2)\,\mathrm{mg\ solubilized\ sludge\ (mg\ sludge)}^{-1}$ $\beta = 1.2\,\mathrm{mg\ COD}\left(\mathrm{mg\ MLSS}^{-1}\right)$, $\mu_m = 1 - 10\,(1)\,\mathrm{day}^{-1}$. Where a range of values is stated, we provide in parenthesis the standard value used for our calculations.

The corresponding values of the dimensionless parameters are: $S_0^* = 20 - 400(40)$, $k_d^* = 0.0028 - -0.028(0.028)$, $k_h^* = 0.006 - -2.2(0.3)$, $\beta^* = 0.6$, $\tau^* = 4 - -40(4)$.

An easy calculation shows that this range for τ^* is always greater than the washout value.

B Stability calculation for the no-washout solution branch

The Jacobian matrix for the no-washout branch is given by

$$J\left(S^*, X^*, P^*\right) = \begin{pmatrix} -\frac{1}{\tau^*} - \frac{X^*}{\left(1+S^{*2}\right)} & \frac{(\alpha\beta^*-1)D}{\tau^*} - k_d^* & \frac{\alpha\beta^* D}{\tau^*} + \beta^* k_h^* \\ \frac{X^*}{(1+S^*)^2} & 0 & 0 \\ 0 & \frac{(1-\alpha)D}{\tau^*} & -\frac{\alpha D}{\tau^*} - k_h^* \end{pmatrix}, \tag{27}$$

where we have simplified the expressions at $J(1,2)$ and $J(2,2)$ using the relationship that along the no-washout branch

$$\frac{-D}{\tau^*} + \frac{S^*}{1+S^*} - k_d^* = 0. \tag{28}$$

We now define

$$A = \frac{X^*}{\left(1+S^*\right)^2}, \qquad B = \frac{\alpha D}{\tau^*} + k_h^*, \qquad C = \frac{\left(\alpha\beta^* - 1\right)D}{\tau^*} - k_d^*, \qquad D = \frac{(1-\alpha)}{\tau^*}.$$

Note that for physically meaningful solutions we have $A > 0$, $B > 0$, $C < 0$ and $D \geq 0$.

The characteristic polynomial of the Jacobian matrix (27), $\mathcal{C}\left(\lambda\right)$, is given by

$$\begin{aligned} \mathcal{C}\left(\lambda\right) &= \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3, \\ a_1 &= \frac{1 + \left(A+B\right)\tau^*}{\tau^*}, \\ a_2 &= \frac{\left(B-C\right)A\tau^* + B}{\tau^*}, \\ a_3 &= -AB\left(C + \beta^* D\right). \end{aligned}$$

By the Routh-Hurwitz theorem the real parts of the eigenvalues of the characteristic equation $\mathcal{C}\left(\lambda\right)$ have negative real part when

$$a_1 > 0, \qquad a_1 a_2 - a_3 > 0, \qquad a_3 > 0.$$

By inspection $a_1 > 0$. Consider the condition $a_3 > 0$. We have

$$\begin{aligned} a_3 &= -AB\left(C + \beta^* D\right), \\ &= -AB\left(\frac{\left(\alpha\beta^* - 1\right)}{\tau^*} - k_d^* + \frac{\beta^*\left(1-\alpha^*\right)}{\tau^*}\right) \qquad \text{(using the definition of } C \text{ and } D\text{)}, \\ &= -AB\left(\frac{\beta^* - 1}{\tau^*} - k_d^*\right) \\ &> 0 \qquad \text{as } AB > 0,\ 0 \leq \beta^* \leq 1 \text{ and } k_d^* > 0. \end{aligned}$$

After some algebra the expression $a_1 a_2 - a_3 > 0$ can be written as

$$\begin{aligned} a_1 a_2 - a_3 &= \frac{1}{\tau^{*2}}\left[b_2 \tau^{*^2} + b_1\tau^* + b_0\right], \\ b_2 &= AB\left(A + B + \beta D\right) - A^2 C, \\ b_1 &= B\left(2A + B\right) - AC, \\ b_0 &= B. \end{aligned}$$

As $A > 0$, $B > 0$, $C < 0$, $D \geq 0$ and $\tau^* > 0$ it immediately follows that $a_1 a_2 - a_3 > 0$. Thus the no-washout branch is stable whenever it is physically meaningful.

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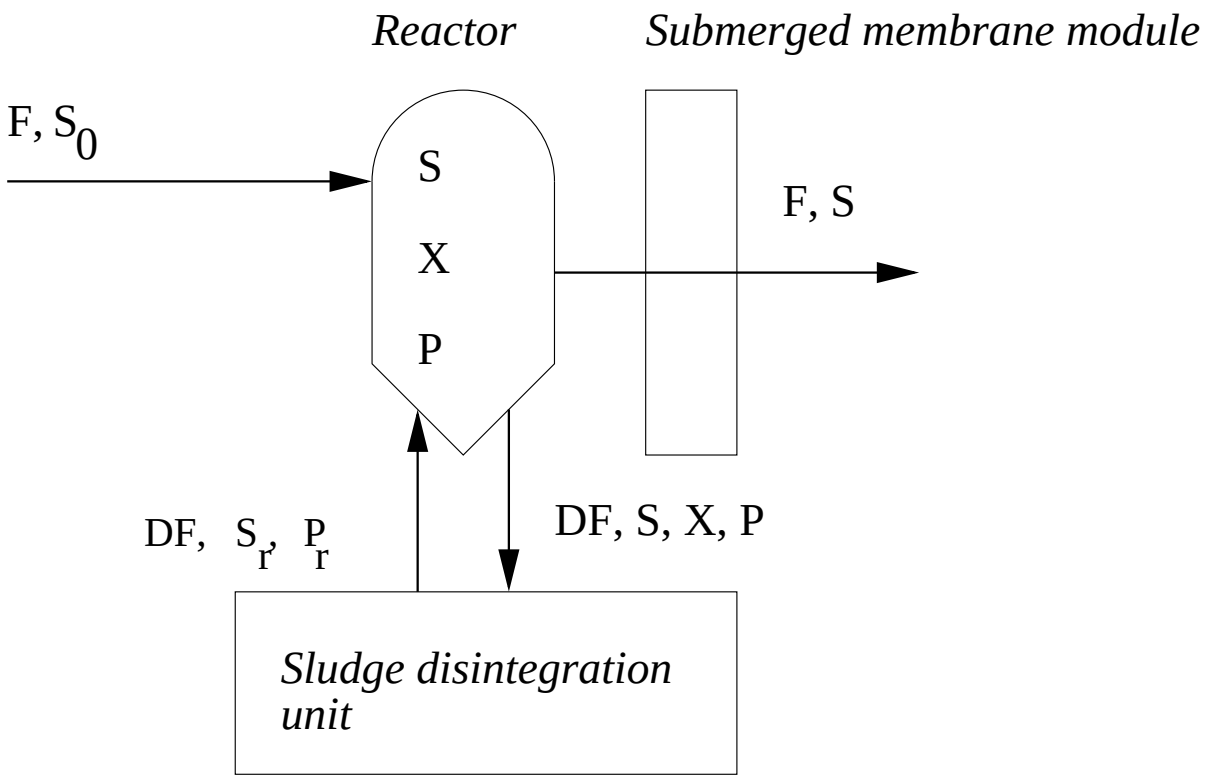
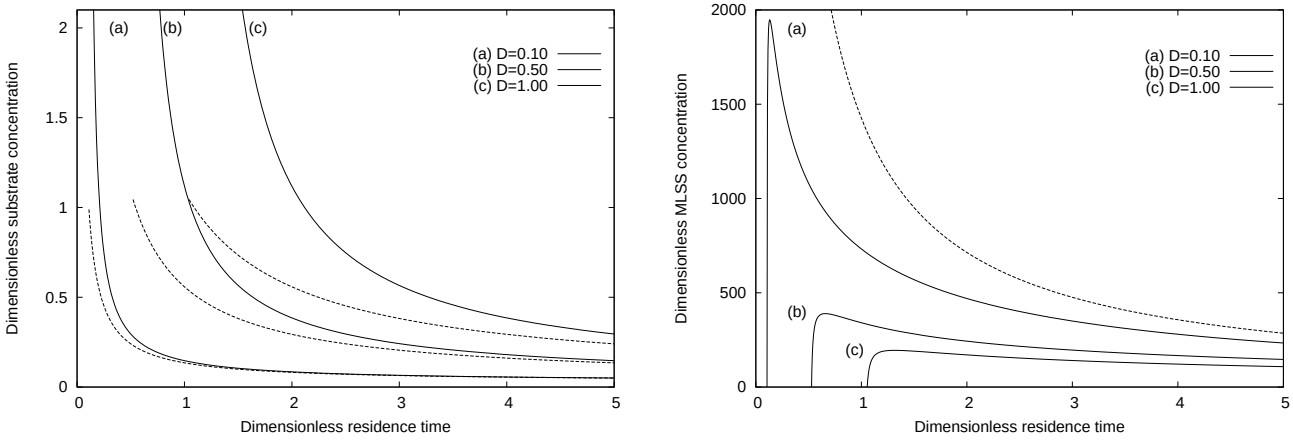


Figure 1: Schematic figure of a membrane bioreactor with a sludge disintegrator unit. The volume flux through the reactor is F, with DF diverted through the SDU. The substrate concentration in the feed is S_0. The SDU returns substrate and product concentrations S_r and P_r to the reactor. After Yoon (2003).



(a) (b)

Figure 2: Steady-state diagrams showing the variation of substrate concentration (S^*) and MLSS as a function of the residence time (τ^*). The dashed lines are the asymptotic approximation for large values of the dimensionless residence times to order $1/\tau^*$. Parameter values: sludge solubilization ratio; $\alpha = 0.2$; dimensionless particulate to substrate conversion factor, $\beta = 0.6$; dimensionless death rate, $k_d^* = 0.028$; dimensionless hydrolysis date, $k_h^* = 0.3$, dimensionless feed concentration, $S_0^* = 40$. The value of the sludge disintegration factor (D) is given in the figure key.

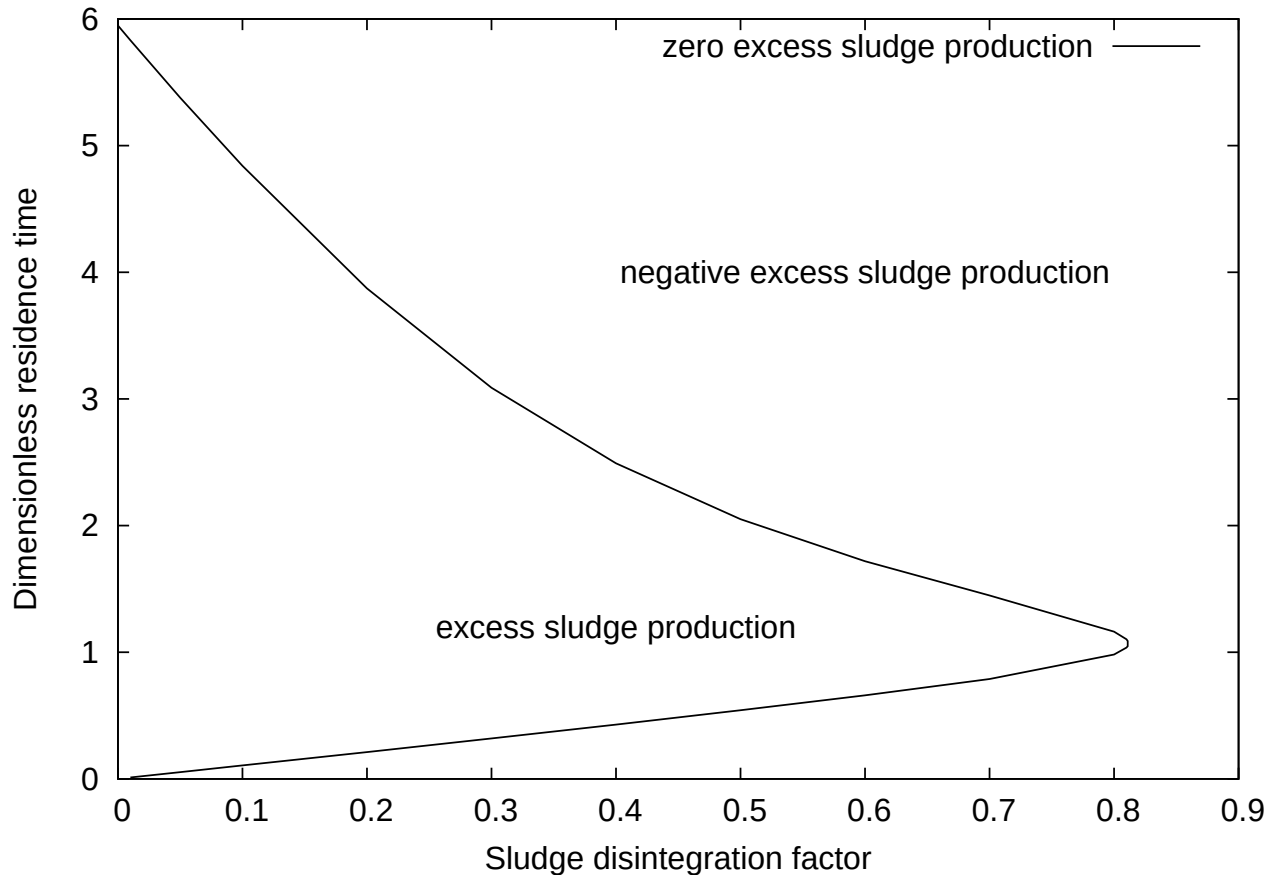


Figure 3: The zero excess sludge production line as a function of sludge disintegration rate (D) and dimensionless residence time (τ^*). This divides the plane into regions of 'excess' and 'negative excess' sludge production. Target value: $\mathrm{MLSS}^*_{\mathrm{target}} = 240$.